0907542 Patter Recognition (Spring 2010)
Midterm Exam

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Instructions: Time **60** min. Closed books & notes. No calculators or mobile phones. No questions are allowed. Show your work clearly. Every problem is for 10 marks.

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Q1. Consider a two-class two-dimensional classification task, where the feature vectors in each of the classes ω_1 and ω_2 are distributed according to

$$p(x|\omega_1) = \frac{1}{\left(\sqrt{2\pi\sigma_1^2}\right)^2}\exp\left(-\frac{1}{2\sigma_1^2}(x-\mu_1)^T(x-\mu_1)\right)$$

$$p(x|\omega_2) = \frac{1}{\left(\sqrt{2\pi\sigma_2^2}\right)^2}\exp\left(-\frac{1}{2\sigma_2^2}(x-\mu_2)^T(x-\mu_2)\right)$$

with

$$\mu_1 = [1, \quad 1]^T, \quad \mu_2 = [1.5, \quad 1.5]^T, \quad \sigma_1^2 = \sigma_2^2 = 0.2$$

Assume that $P(\omega_1) = P(\omega_2)$ and design a Bayesian classifier that minimizes the error probability.

Solution:

$$p(x|\omega_1)P(\omega_1) >< p(x|\omega_2)P(\omega_2)$$

$$p(x|\omega_1) >< p(x|\omega_2)$$

$$\frac{1}{\left(\sqrt{2\pi\sigma_1^2}\right)^2}\exp\left(-\frac{1}{2\sigma_1^2}(x-\mu_1)^T(x-\mu_1)\right) >< \frac{1}{\left(\sqrt{2\pi\sigma_2^2}\right)^2}\exp\left(-\frac{1}{2\sigma_2^2}(x-\mu_2)^T(x-\mu_2)\right)$$

$$\exp\left(-\frac{1}{2\sigma_1^2}(x-\mu_1)^T(x-\mu_1)\right) >< \exp\left(-\frac{1}{2\sigma_2^2}(x-\mu_2)^T(x-\mu_2)\right)$$

$$\left(-\frac{1}{2\sigma_1^2}(x-\mu_1)^T(x-\mu_1)\right) >< \left(-\frac{1}{2\sigma_2^2}(x-\mu_2)^T(x-\mu_2)\right)$$

$$\frac{1}{2\sigma_1^2}(x-\mu_1)^T(x-\mu_1) <> \frac{1}{2\sigma_2^2}(x-\mu_2)^T(x-\mu_2)$$

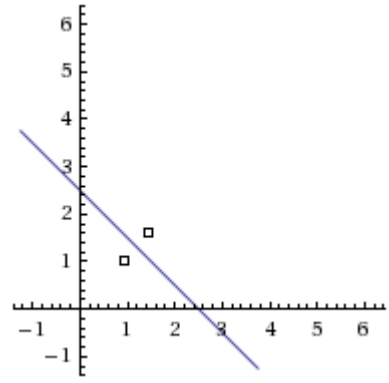
$$(x-\mu_1)^T(x-\mu_1) <> (x-\mu_2)^T(x-\mu_2)$$

$$\begin{bmatrix}x_1-1\\x_2-1\end{bmatrix}^T\begin{bmatrix}x_1-1\\x_2-1\end{bmatrix} <> \begin{bmatrix}x_1-1.5\\x_2-1.5\end{bmatrix}^T\begin{bmatrix}x_1-1.5\\x_2-1.5\end{bmatrix}$$

$$(x_1-1)^2+(x_2-1)^2 <> (x_1-1.5)^2+(x_2-1.5)^2$$

$$-2x_1+1-2x_2+1 <> -3x_1+2.25-3x_2+2.25$$

$$x_1+x_2 <> 2.5$$



Q2. Consider a case in which class ω_1 consists of two feature vectors $[0, \quad 0]^{\boldsymbol{T}}$ and $[0, \quad 1]^{\boldsymbol{T}}$ and class ω_2 of $[\mathbf{1}, \quad \mathbf{0}]^{\boldsymbol{T}}$ and $[\mathbf{1}, \quad \mathbf{1}]^{\boldsymbol{T}}$. Use the perceptron algorithm in its form shown below, with $\rho_t = 0.7$ and $\boldsymbol{w}(0) = [-0.4, \quad 1 \quad 1]^T$, to design the line separating the two classes. Draw the samples and the resulting classification line.

$$\boldsymbol{w}(t+1) = \boldsymbol{w}(t) - \rho_t \sum_{\boldsymbol{x} \in \boldsymbol{Y}} \boldsymbol{\delta_x x}$$

Solution:

$$\boldsymbol{w}(t+1) = \boldsymbol{w}(t) - \rho_t \sum_{\boldsymbol{x} \in \boldsymbol{Y}} \boldsymbol{\delta_x x}$$

$$\boldsymbol{w}(1) = \boldsymbol{w}(0) - \rho_t \sum_{\boldsymbol{x} \in \boldsymbol{Y}} \boldsymbol{\delta_x x} = \begin{bmatrix} -0.4 \\ 1 \\ 1 \end{bmatrix} - 0.7(+1)\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - 0.7(+1)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1.4 \\ 0.7 \\ 1.4 \end{bmatrix} = \begin{bmatrix} -1.8 \\ 0.3 \\ -0.4 \end{bmatrix}$$

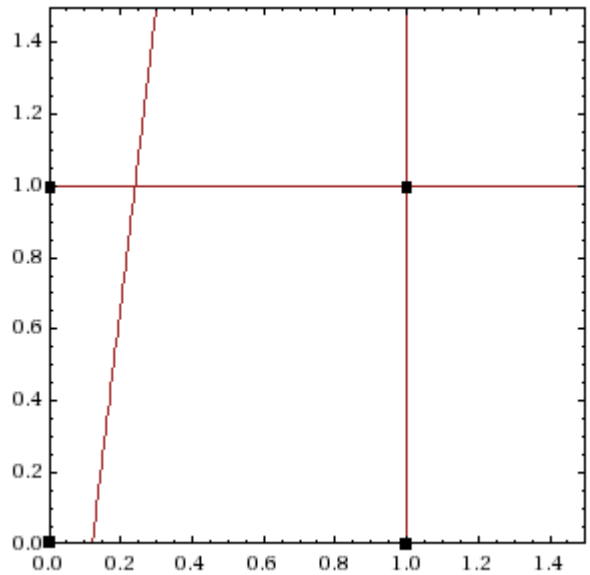
$$\boldsymbol{w}(2) = \boldsymbol{w}(1) - \rho_t \sum_{\boldsymbol{x} \in \boldsymbol{Y}} \boldsymbol{\delta_x x} = \begin{bmatrix} -1.8 \\ 0.3 \\ -0.4 \end{bmatrix} - 0.7(-1)\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 0.7(-1)\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.8 \\ 1 \\ 1 \end{bmatrix}$$

$$\boldsymbol{w}(3) = \begin{bmatrix} -1.8 \\ 1 \\ 1 \end{bmatrix} - 0.7(+1)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 0.3 \\ 0.3 \end{bmatrix}$$

$$\boldsymbol{w}(3) = \begin{bmatrix} -1.8 \\ 1 \\ 1 \end{bmatrix} - 0.7(+1)\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.5 \\ 0.3 \\ 0.3 \end{bmatrix}$$

$$\boldsymbol{w}(3)^t \boldsymbol{x} = 0$$

$$-2.5x_1 + 0.3x_2 + 0.3 = 0$$



Q3. You are considering building a classifier using two of the three features shown in the table below. Given the nine samples shown in this table, select two features using the *scatter criterion* method.

Sample	x_1	x_2	x_3	Class
1	5.5	4	31	1
2	5	4	35	1
3	6	1	33	1
4	3	0	27	2
5	1	0	25	2
6	2	3	23	2
7	7.5	5	40	3
8	8	5	50	3
9	7	5	60	3

					Mean			Diff *Diff			Within			between		
Sample	x1	x2	x3	Class	x1	x2	x3	x1	x2	x3	x1	x2	x3			
1	5.5	4	31	1	5.5	3	33	0	1	4	0.167	2	2.667	0.25	0	9
2	5	4	35	1	5.5	3	33	0.25	1	4						
3	6	1	33	1	5.5	3	33	0.25	4	0						
4	3	0	27	2	2	1	25	1	1	4	0.667	2	2.667	9	4	121
5	1	0	25	2	2	1	25	1	1	0						
6	2	3	23	2	2	1	25	0	4	4						
7	7.5	5	40	3	7.5	5	50	0	0	100	0.167	0	66.67	6.25	4	196
8	8	5	50	3	7.5	5	50	0.25	0	0						
9	7	5	60	3	7.5	5	50	0.25	0	100						
					5	**3**	**36**				**1**	**4**	**72**	**15.5**	**8**	**326**
										J	**16.5**	**3**	**5.528**			

Select x1 and x3.

<Good Luck>